ECE 536 – Spring 2022

Homework #7 – Solutions

Problem 1)

(A) X=0.37 First, we need to find the lattice constant using linear interpolation. That is

$$a(x) = a(GaAs)x + a(InAs)(1-x) = 5.6533x + 6.0584(1-x)$$

This gives a lattice constant of 5.8688A, leading to $\epsilon_{xx} = \epsilon_{yy}$ =-0.006721. Using the same linear interpolation to find the vales for C_{12} and C_{11}, we find $\epsilon_{zz} = 0.006748$. Apply the linear interpolation a third time to find the deformation potentials $a_v = 1.059eV$, $a_c = -5.853eV$, and $b = -1.763eV$. Finally,

$$\begin{aligned} P_\epsilon &= -a_v(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) = 0.007901eV \\ Q_\epsilon &= \frac{-b}{2}(\epsilon_{xx} + \epsilon_{yy} - 2\epsilon_{zz}) = -0.02375eV \end{aligned}$$

These give the final results for the band offsets at zone center

$$\begin{aligned} E_c &= E_g + a_c \mathrm{Tr}\epsilon = 0.6769eV \\ E_{hh} &= -P_\epsilon - Q_\epsilon = 0.01666eV \\ E_{lh} &= -P_\epsilon + Q_\epsilon = -0.03084eV \\ E_{so} &= -P_\epsilon - \Delta = -0.3723eV \end{aligned}$$

(B) X=0.57 Follow the same procedure as above (just changing the mole fraction) to arrive at the band offsets at zone center as

$$\begin{aligned} E_c &= E_g + a_c \mathrm{Tr}\epsilon = 0.8071eV \\ E_{hh} &= -P_\epsilon - Q_\epsilon = -0.01633eV \\ E_{lh} &= -P_\epsilon + Q_\epsilon = 0.03230eV \\ E_{so} &= -P_\epsilon - \Delta = -0.3492eV \end{aligned}$$

The plots are shown in Fig. 1.1 against the lattice matched case.

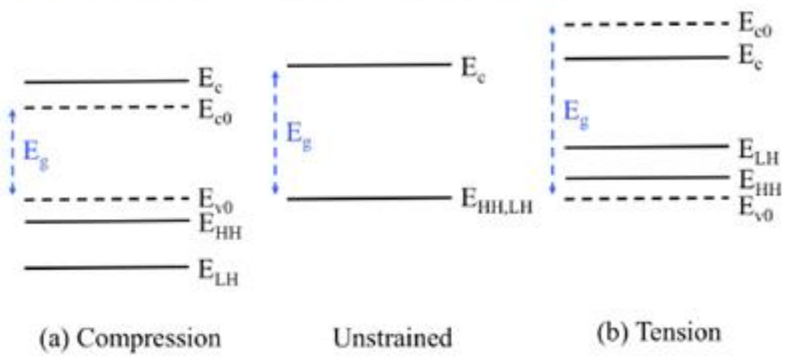

Figure 1.1: Band-Edge diagrams for compressive strain (part a) and tensile strain (part b) plotted with respect to the unstrained case, shown in the center.

Problem 2)

(A) THRESHOLD VS. LENGTH The increase of J_{th} with inverse cavity length can be explained by the the fact that decreasing the cavity length will increase the threshold material gain by increasing the mirror loss. By finding the rate of that change, we see that $\ln J_{th}$ varies linearly with inverse cavity length as

$$L_{opt} = \frac{1}{2}\frac{1}{n_W \Gamma_W g_0} \ln \frac{1}{R_1 R_2}$$

(B) MINIMIZING THRESHOLD CURRENT Using the relation above, given a particular number of quantum wells, the optimal cavity length can be found. This will result in the minimum threshold current.

(C) PLOTS OF I_{th} For this problem, we will assume that $J_0 = 200\text{A/cm}^2$, $g_0 = 3000\text{cm}^{-1}$, Γ_W is about 2%, and the intrinsic loss is about 20 per cm. In addition, the mirror reflectivities are taken to be 30%, the carrier injection is about 0.8, and the width is taken to be 2μm.

The log of the threshold current density is given as

$$\ln J_{th} = \ln \frac{n_w J_0}{\eta} + \frac{\alpha}{n_W \Gamma_W g_0} + \frac{L_{opt}}{L} - 1$$

and is shown in Fig. 2.2. Using the same relationship and casting the threshold current density in terms of the threshold current ($I_{th} = J_{th} wL$), we can plot threshold current as a function of cavity length. That plot is shown in Fig. 2.1.

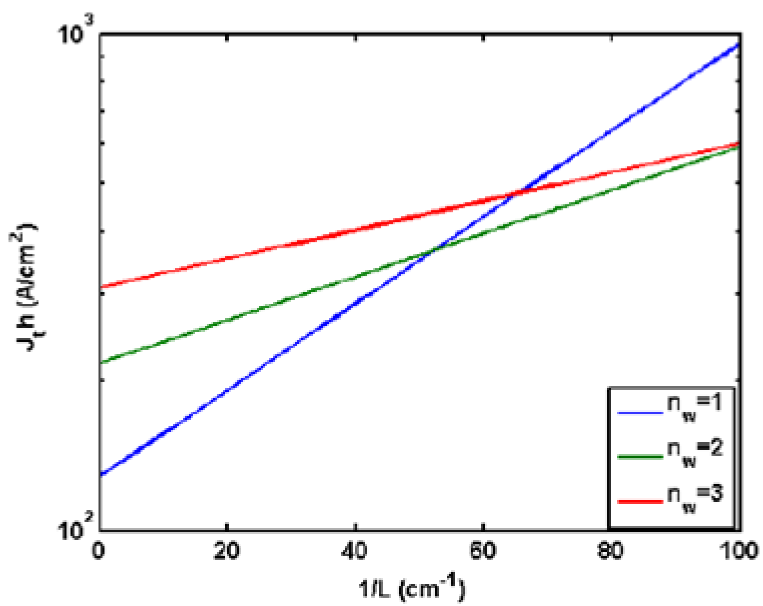

Figure 2.1: Threshold current density vs. inverse cavity length.

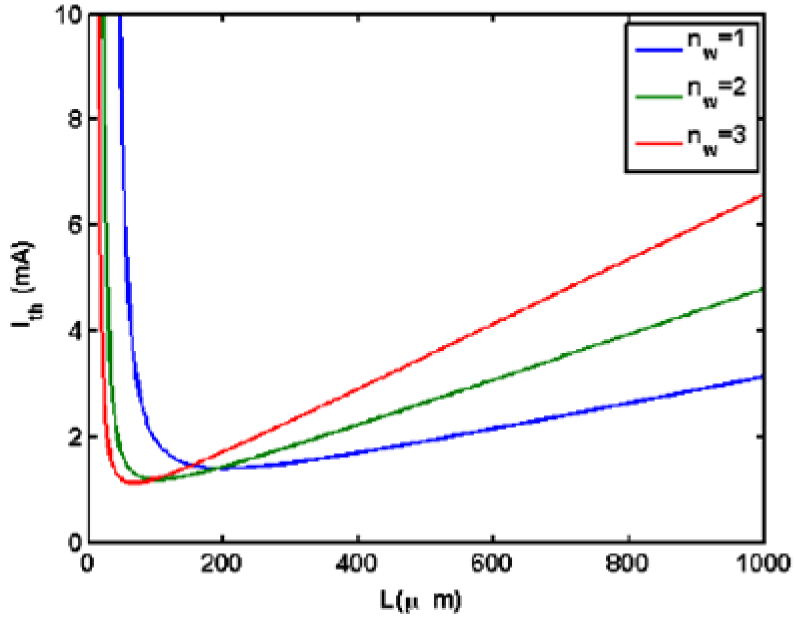

Figure 2.2: Threshold current vs. cavity length.

Problem 3)

Before considering any of the QWs themselves, its best to start with the barrier. From the example on page 791 of the text using Model-Solid Theory, we can write the average valence band energy of the barrier in absolute terms as

$$E^B_{v,av} = xE_{v,av}(\text{InAs}) + (1-x)E_{v,av}(\text{AlAs}) + 3x(1-x)\frac{\Delta a}{a}\left[a_v(\text{InAs}) - a_v(\text{AlAs})\right]$$

which gives -6.9889eV. Then using a weighted average, we find Δ^B to be 0.332eV. Then, since

$$E^B_v = E^B_{v,av} + \frac{\Delta^B}{3} = -6.8782\text{eV}$$

we can find that E^B_c =-5.442eV.

(A) X=0.37 Using linear interpolation for the the material parameters we find

1. a_W =5.9085A
2. $C_{11} = 9.6425\times10^{11}$dyne/cm^2, C_{12} =4.8405$\times10^{11}$dyne/cm^2
3. $a_v = 1.0592$eV, $a_c = -5.8533$eV

4. $b = -1.763\text{eV}$

5. $\Delta^W = 0.3652\text{eV}$

which leads directly to the strain parameters $\epsilon_{xx} = \epsilon_{yy}$ =-6.998×10^{-3} and $\epsilon_{zz} = 7.026 \times 10^{-3}$. Therefore, $\delta E_c = a_c \text{Tr}\epsilon = 40.8\text{meV}$, $P_\epsilon = 7.4\text{meV}$, and $Q_\epsilon = -24.7\text{meV}$. The average valence band offset and band gap are calculated in the same way as the barrier case above to be -6.7545eV and 0.6378eV, respectively.

Therefore, we find that the original valence band level $E_v^{W,0} = E_{v,av} + \Delta^W/3 = -6.6327\text{eV}$ and $E_c^{W,0} = -5.9950\text{eV}$. Then, to find the conduction band including strain, $E_c^W = E_c^{W,0} + \delta E_c = -5.9542\text{eV}$. Likewise, $E_{HH}^W = E_v^{W,0} - P_\epsilon - Q_\epsilon = -6.6154\text{eV}$ and $E_{LH}^W = E_v^{W,0} - P_\epsilon + Q_\epsilon = -6.6648\text{eV}$. Finally, finding the energy levels with respect to the valence and conduction bands

1. $\Delta E_c = E_c^B - E_c^W = 532.3\text{meV}$

2. $\Delta E_{HH} = E_{HH}^W - E_{HH}^B = 262.8\text{meV}$

3. $\Delta E_{LH} = E_{LH}^W - E_{LH}^B = 213.4\text{meV}$

The energy levels are plotted in Fig. 3.1. Note that this sample is compressively strained, pushing the heavy hole band above the light hole band and shrinking the band gap.

(B), (C) These are done in an analogous way to part (a) above and the results are shown in Fig. 3.1. Note that part (b) is unstrained and the two hole band energies are degenerate at zone center. Part (c) is under tensile strain, widening the band gap and pushing the light hole band above the heavy hole band at zone center.

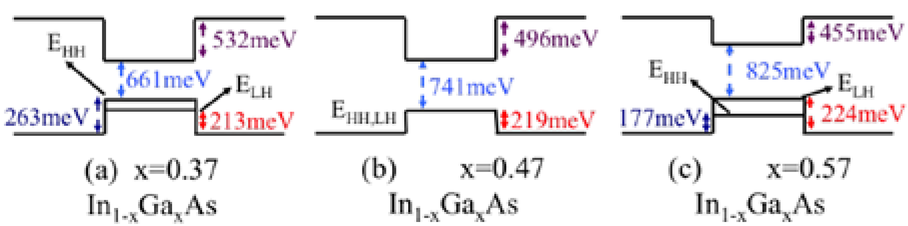

Figure 3.1: Band-Edge plots for different mole fractions of InGaAs/InAlAs quantum wells.